"Asymptotic Robustness of DG Schemes for the Convection Diffusion Equation"

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Abstract

Numerical schemes for problems involving both transport and diffusion often frequently fail when the diffusion coefficient is small and the convective velocity is non-smooth. In this talk, we show that schemes using discontinuous Galerkin (DG) approximations of the elliptic (diffusion) term and classical upwinding for the convective term exhibit asymptotic robustness; that is, solutions converge strongly in $L^2(\Omega)$ independently of how the diffusion constant and mesh parameters tend to zero. The major technical difficulty involves identifying the trace (boundary values) of weakly convergent approximations. This is done using Boyer's (2005) extension of the DiPerna Lions theory for transport equations.